Iowa Precalculus Advisory Council
Position Paper on Instruction for Exponential and Logarithmic Functions

As with its statement on trigonometry instruction, the council emphasizes the important of understanding a few topics at great depth. To this end, it proposes that with regards to exponential functions, the fundamental understandings necessary for success in calculus are as follows.

There are two families of functions to be understood: the exponential functions and the logarithmic functions. The logarithmic functions are to be understood as the inverses of the exponential functions. The key feature of exponential functions to be grasped is that they model situations where the growth is governed by a constant *factor* as opposed to linear functions where the growth is by a constant amount. This factor may be a growth factor (in the case of increasing exponential functions) or a decay factor (in the case of decreasing exponential functions). It is important that students be able to distinguish growth/decay factors from growth/decay rates and be able to convert one into the other.

More specifically, the council sets forth the following as fundamental understandings for success in calculus.

1. *Recognizing exponential growth or decay in the context of a word problem.* Students should be able to determine if a situation described in a problem represents exponential growth or decay and form an appropriate mathematical model based on identifying the factor. They should be able to distinguish between linear and exponential growth in these contexts. e.g. "The candle decreases by a third every hour." vs. "The candle decreases by a third of an inch every hour."

2. *Growth/decay factors and rates.* For example, when told that the amount of medicine in a patient's blood decreases by 2.3% an hour, students should be able to determine that the number of milligrams of medicine decreases by a factor of 0.977 every hour and explain why their answer is correct. Hence decay *rate* of 2.3% per hour is equivalent to a decay *factor* of 0.977.

3. 1 *unit and* n *unit growth factors and rates.* For example, if a population of bacteria grows by a factor of 1.1 every month, students should be able to determine that each year it grows by a factor of $(1.1)^{12} \approx 3.138$

and that each year it grows at a rate of $1.1^{12} - 1 \approx 214\%$. Or given a one week growth factor, students should be able to find the corresponding 1-day growth factor and explain why their answer is correct.

4. *Compounding periods* Given an annual percentage rate (APR) of a compounding rate, students should be able to find the corresponding growth rates and factors for other time periods such as monthly, weekly, etc.. Understanding such computations is important for many financial decisions but also sets the stage for understanding continuous exponential growth and decay.

5. *Continuous exponential growth/decay* Continuous exponential growth should be understood as the limit of discrete exponential growth as number of compounding periods grows without bound. The base e arises naturally in this way. In addition, students should be able to distinguish between continuous and discrete exponential growth in the context of a word problem and produce the appropriate model. For example, "The number of milligrams of medicine in the blood decreases by 2.3% an hour" leads to the model $f(t) = A(0.977)^t$ whereas "The number of milligrams of medicine in the blood continuously decreases by 2.3% an hour" leads to the model $f(t) = Ae^{0.023t}$. This should be based on the ability to produce a growth or decay factor from a continuous rate, e.g., in the latter example the hourly decay factor is $e^{0.023}$.

6. *Logarithms* As mentioned above, logarithmic functions should be understood as inverses of the corresponding exponential functions. From this understanding, students should be able to recognize the domain and range of logarithmic functions. It is important to know the commonly used properties of logarithm such as $\ln(ab) = \ln(a) + \ln(b)$ whenever $a, b > 0$. However, students should be able to explain where these properties come from in terms of the corresponding properties of exponents. Finally, students should be able to create models of exponential growth and use log functions to determine when the exponentially growing/decaying quantity reaches a specified amount. e.g. Suppose a savings account has 5% APR compounded monthly and $2000 is invested. How long until balance reaches $100,000?