IPAC Position Paper on Trigonometry Instruction

The purpose of this document is to bring focus to the understandings in trigonometry that are essential for success in calculus.

The amount of trigonometry that must be mastered in order to ensure success in calculus is not great; but it must be understood at a deep level and retained. As in most topics, it benefits the student more to study a few topics in great depth than the excessive memorization of a voluminous canon of less useful topics. The only way to test depth of understanding is to require frequent correct explanations for *why* solutions are correct beyond 'because that's the rule'.

The key trigonometry topics and understandings necessary for success in calculus are as follows.

1. **Radian measure** A strong conceptual understanding of radian measure is essential to understand polar coordinates. Standard formulas for the derivatives of the sine and cosin e functions depend on radian measure. Students must conceive of radian measure in terms of:

   (a) Measuring the subtended arc (from the initial side to the terminal side) when the vertex is placed at the origin.

   (b) Using the radius as the unit of measure.

   (c) Determining the subtended arc's *fraction* of the circumference.

   Multiplying by $\frac{\pi}{180^\circ}$ is all too often the only, and an entirely insufficient, conception of radian measure.

2. **The sine, cosine, and tangent functions in terms of circular motion** Students must understand the sine, cosine, and tangent functions as *functions* (i.e. as input-output processes).

   (a) $\sin(\theta)$ must be understood as the displacement from the horizontal diameter (i.e. distance above the horizontal diameter) of a point on the circumference that results from a rotation of θ radians from the 3 o'clock position on a circle and that this displacement is measured in *radians* (i.e. the radius is used as the unit of measure). See Figure 1.

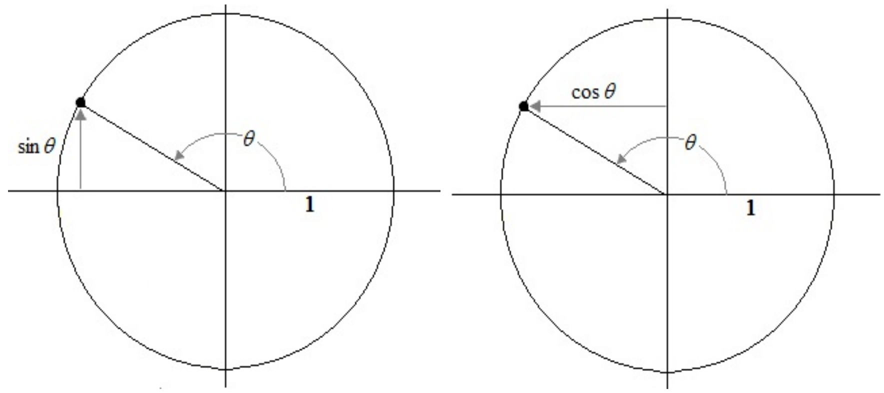

Figure 1: The sine and cosine functions

(b) $\cos(\theta)$ must be understood as the displacement from the vertical diameter (i.e. distance to the right of the vertical diameter) of a point on the circumference that results from a rotation of θ radians from the 3 o'clock position on a circle and that this displacement is measured in *radians* (i.e. the radius is used as the unit of measure). See Figure 1.

(c) $\tan(\theta)$ must be understood as the slope of the radius after a rotation of θ radians from the 3 o'clock position on a circle.

Typical student conception of the sine, cosine, and tangent functions goes no further than 'SOHCAHTOA' which is inadequate for calculus.

3. **Modeling** Students should be able to demonstrate their understanding of the sine, cosine, and tangent functions through modeling. For example, by constructing an expression for the distance from the ground in meters of a point on a Ferris wheel t seconds after it has started, given the radius of the Ferris wheel and the distance of its lowest point from the ground.

Students should be able to demonstrate their understanding of the above topics by using them to explain their work on other topics such as triangle trigonometry, principal values, identities, and solving equations. In particular:

- Identities should be focused on a small number (around 5) that are essential for calculus.

- While we recognize that most students entering a precalculus or trigonometry course will have first seen right-triangle trigonometry, it is essential that they eventually develop the understanding of right triangle trigonometry as a particular case of circular trigonometry.

- By connecting their understanding of trigonometric functions to the function concept, students will be able to manipulate trigonometric functions as components of algebraic expressions.

- Students should be able to solve straightforward trigonometry equations and identify among the infinitely many solutions those that are meaningful.

- Students should be able to construct exact expressions for the values of the trigonometric functions on the reference angles ($\pi/4$, $\pi/3$, $\pi/6$) and be able to extend them to values in other quadrants.