Summary of Essential Content to Prepare Students for Calculus

Statement of Purpose.

The purpose of this document is to summarize the most essential mathematical content that every student must learn in order to be successful in first- and second-semester calculus. To "learn" means to take possession of the knowledge so that the student can apply it, unprompted, in new or even unfamiliar settings.

Common Core Citations	
	Number Systems and Arithmetic
	1. Understand Fundamental Properties of Numbers.
	2. Perform Addition, Subtraction, Multiplication, Division, Exponentiation with
N-RN 3	* Integers
	* Rational Numbers
N-RN 2	* Irrational Numbers
	In particular: i) understand operations applied to radicals
	ii) be comfortable with π, e, square roots
	Properties of Expressions
	1. Understand fundamental properties of expressions.
A-SSE 2, **A-SSE 3,** **F-IF 8a**	2. Perform standard factoring and expansion formulae and techniques:
	* trinomials
	* perfect squares
A-SSE 1,2	* difference of squares
	* grouping
N-RN 2	* and use as structural templates for arbitrary f and g.
	* perform polynomial long division (the *Division Algorithm*)
	3. Know laws of exponents and radicals.
F-LE 8	4. Know definition and properties of logarithms.
	Laws of Exponents and Radicals
	Laws of Logarithms
A-APR 5	**Properties of Equality**
	Properties of inequalities (Know the properties of inequalities.)
A-CED 2	**Rectangular Coordinate Plane**
	1. Vocabulary:
	* horizontal axis ("x-axis"), vertical axis ("y-axis")
	* quadrants I, II, III, IV
	* ordered pair: (x, y), (a, b)
	* origin: $(0, 0)$

Comment [LS3]: I agree with N-RN 3 for the above part but I think this aligns with N-RN 2

Comment [LS4]: Aligns with N-RN 2

Comment [LS5]: Aligns with F-LE 8

G-GPE 7	* relation: Any set of ordered pairs.
	* graph: The plotting of a relation.
	* domain: The set of first coordinates of a relation.
	* range: The set of second coordinates of a relation.

2. Distance Formula

3. Pythagorean Theorem

4. Understand and use the geometry/algebra of lines in 2-dimensions:

G-GPE 5,
G-CO 1

    * slope

    * point/slope and slope/intercept forms for a line.

    * parallel

    * perpendicular

F-IF 1
F-IF 9

G. <u>Functions</u>

1. Know that a function is a rule that assigns to each input exactly one output

F-BF 3

2. Understand what are Domains and Ranges, and be able to find them.

3. Perform transformations of functions:

    * horizontal/vertical shifts

    * stretching/compressing

    * reflections

F-BF 3
F-IF 7b
F-IF 1,2, 3

4. Be able to examine functions for symmetry.

5. Be able to read and construct Piecewise Functions.

6. Read/use the algebraic notation of functions, their operations and compositions:

    * Special structures:

        * $g(x) = \left| f(x) \right|$, i.e. what this particular composition produces

F-IF 6

        * Difference Quotient

        * Average rate of change in f over an interval $[a, b]$

F-BF 4

7. Understand inverse functions:

    * Understand the property "one-to-one", and what it implies.

    * Find and use the inverse of f

    * Contrast the above and its notation to the reciprocal of f

F-TF 1
F-TF 1

8. Some Trigonometric Particulars to know

    * DMS / radians

F-TF 2, 3

    * the *Unit Circle* and its role in defining trigonometric objects

    * sine and cosine values for reference angles

        (multiples of fragments of π)

    * Definitions for each ratio

    * Some Fundamental Trigonometric Identities:

        * Pythagorean identities

        * Even/odd identities

        * Sum/difference identities for sine and cosine

        * Double-angle identities for sine and cosine

Standards	Content

F-IF 4
F-IF 5
F-TF 5
F-TF 7
	9. Applications
	    * Have experience relating and using functions and their properties in applied settings.
	    * Have experience interpreting slopes as rates of changes in applied settings.

	<u>Solving Equations</u>

F-IF 8
F-IF 8
A-REI 4ab
A-SSE
A-APR 6
A-APR 2,3
	1. Fundamental facts/skills:
	    * Zero Product Property
	    * Square Root Property
	    * Completing the square . . . and its offspring, the "quadratic formula"
	    * The Factor Theorem
	    * The Linear Factorization Theorem
	    * Equivalence of exponents, *i.e.*, $b^x = b^y \leftrightarrow x = y.$

A-REI 1,3
A-REI 4,
N-CN 7
A-APR 2,3
N-CN 9
A-REI 2
	2. Equation types and their solutions:
	    * Linear – can always solve.
	    * Quadratic – can always solve.
	    * Higher-degree polynomial – requires a bit of theory
	    * Rational
	    * Radical

F-BF 5 **F-BF 5**	* Exponential – often requires $b^x = y \Leftrightarrow \log_b y = x$ * Logarithmic – often requires $b^x = y \Leftrightarrow \log_b y = x$
F-TF 5	* Trigonometric – solutions are always periodic (if no restrictions)
	H. <u>Geometric Formulae</u>
	1. Some fundamental facts and formulae that consistently appear in the use of previous topics:

G-GPE 7
G-GMD 1
	* Lengths
	        * perimeters
	        * circumference

G-GPE 7
G-GPE 7
G-GMD 1
	* Areas
	        * rectangular
	        * triangular
	        * circular

G-GMD 1
G-GMD 1
	* Volumes
	        * rectangular box
	        * prism, or any solid with congruent cross-sections
	        * sphere

Comment [LS6]: I think you could add F-IF4, 5 and F-TF 5, 7 to this

Comment [LS7]: F-TF 5

Table: Function Families and What to Know

Function Family	Domain	Range	Graph/Shape	Continuity	Intercepts	Symmetry	Continuity	Asymptotes	Endpoint Behavior	Inverse	Division Algorithm	Avg Rate of Change	Growth/Decay	Amplitude	Period	Phase Shift
Constant $f(x)=k$	√	√	√	√	√	√	√		√			√				
Linear $f(x)=mx+b$	√	√	√	√	√	√	√		√	√		√				
Polynomial $f(x)=ax^n+\ldots$	√	√	√	√	√	√	√		√	√		√				
Absolute Value $f(x)=|x|$	√	√	√	√	√	√	√		√							
Square Root $f(x)=\sqrt{x}$	√	√	√	√	√	√	√		√	√		√				
Cube Root $f(x)=\sqrt[3]{x}$	√	√	√	√	√	√	√		√	√		√				
nth Root $f(x)=\sqrt[n]{x}$	√	√	√	√	√	√	√	√	√	√		√				
Rational Power $f(x)=x^{n/m}$	√	√	√	√	√	√	√		√	√						
Semicircle $\sqrt{b^2-x^2}$	√	√	√	√	√	√	√		√							
Rational $f(x)=p(x)/q(x)$	√	√	√	√	√	√	√	√	√	√	√	√				
Exponential $f(x)=a^x$	√	√	√	√	√	√	√	√	√	√		√	√			
Logarithmic $f(x)=\log_b x$	√	√	√	√	√	√	√	√	√	√		√	√			
Sine $f(x)=\sin x$	√	√	√	√	√	√	√		√	√		√		√	√	√
Cosine $f(x)=\cos x$	√	√	√	√	√	√	√		√	√		√		√	√	√
Tangent $f(x)=\tan x$	√	√	√	√	√	√	√	√		√		√			√	√
Cotangent $f(x)=\cot x$	√	√														
Cosecant $f(x)=\csc x$	√	√														
Secant $f(x)=\sec x$	√	√														